MATH 3: Exam 1

Problem 1. (10 points) Consider the two functions $f(x)$ and $g(x)$ defined by the following tables:

x	2	3	4	5
$f(x)$	12	5	6	6

x	5	6	7	9	12
$g(x)$	1	2	3	6	13

Find the domain and range of the composite function $(g \circ f)(x)$:
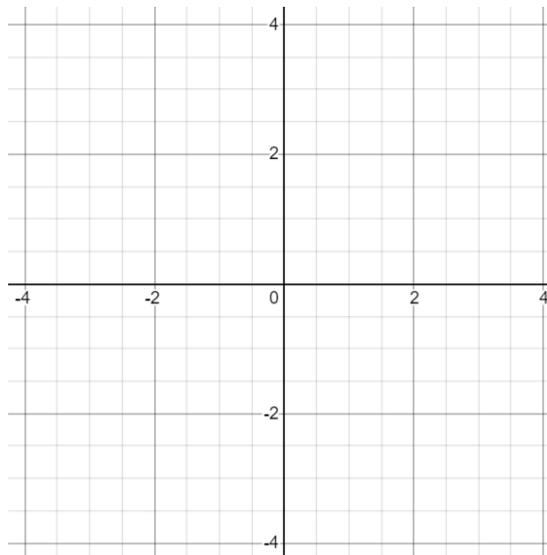
Problem 2. Let $f(x) = \frac{1}{2}x + 1$.

(a) (3 points) Calculate the x-intercept of $f(x)$.

(b) (3 points) Find the equation of the line $g(x)$ that is perpendicular to $f(x)$ and passes through the point $(1/2, 0)$.

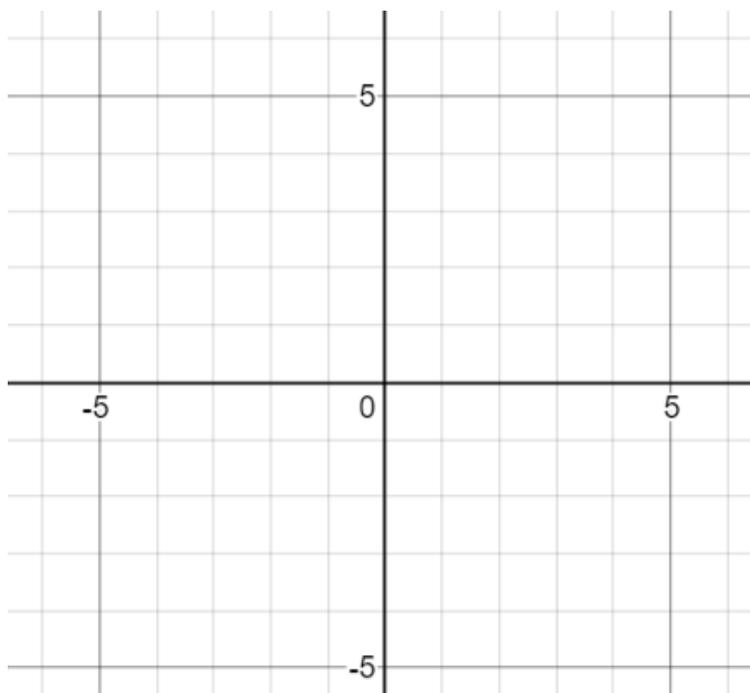(c) (3 points) Calculate the point where $f(x)$ and $g(x)$ intersect.

(d) (3 points) Graph both lines on the grid below:

Problem 3. Let $f(x) = |x|$ and $g(x) = -2|x+1| + 1$.

(a) (5 points) Explain how the graph of $g(x)$ can be obtained from the graph of $f(x)$ using transformations. Make sure to describe the transformations in the correct order!
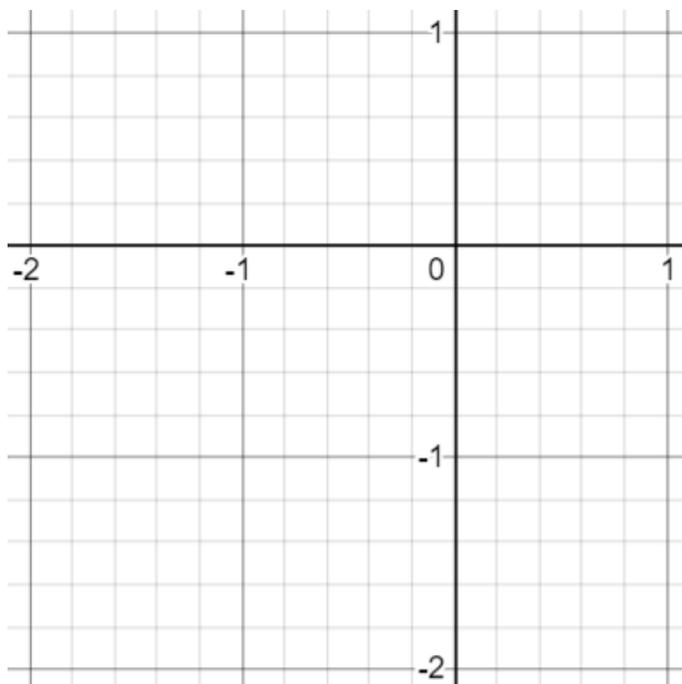
(b) (5 points) Graph $g(x)$ on the grid below.

Problem 4. Consider the quadratic function $f(x) = 3x^2 + 2x - 1$ given in general form.

(a) (3 points) Identify the vertex, the line of symmetry, and the x and y-intercepts of $f(x)$.

(b) (3 points) Identify the range of $f(x)$.

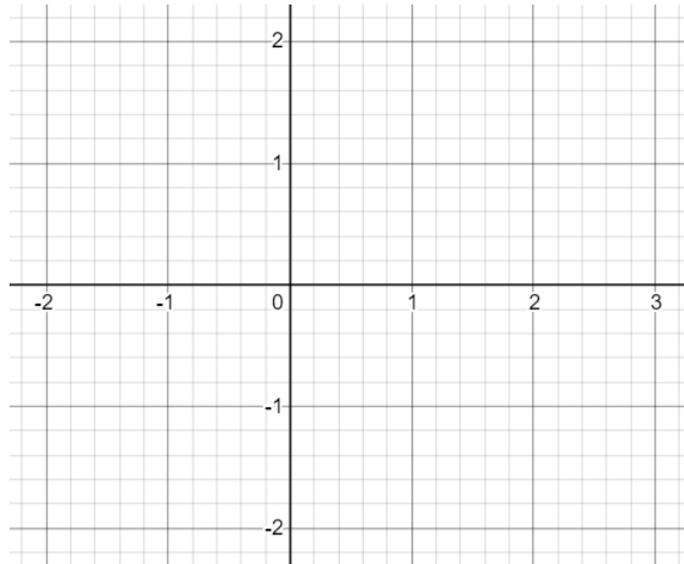(c) (3 points) Write $f(x)$ in standard form.

(d) (3 points) Graph $f(x)$ on the grid below:

Problem 5. Let $f(x) = -\frac{32}{27}(x-1)^3(x+1)$.

(a) (5 points) Identify the degree, the end-behaviour, the zeros and their multiplicities, and the y-intercept of $f(x)$.

(b) (5 points) Graph $f(x)$ on the grid below. Make use the following fact: the point $(-1/2, 2)$ is a turning point for $f(x)$. You may use the approximation $\frac{32}{27} \approx 1.2$.


Problem 6. Ferrell's Donuts in Santa Cruz paid $25,000 in rent, insurance, and other operating expenses in April of 2021. It costs $.50 to produce each donut.

(a) (5 points) Find a linear model $C(d)$ that represents the cost of operating Ferell's donuts in April 2021 as a function of d the number of donuts produced.

(b) (5 points) The revenue in April 2021 was found to be given by the function $R(d) = 3d$. How many donuts did Ferell's donuts need to produce that month in order to make a profit?

Problem 7. (12 points) Consider the following graph of a polynomial.
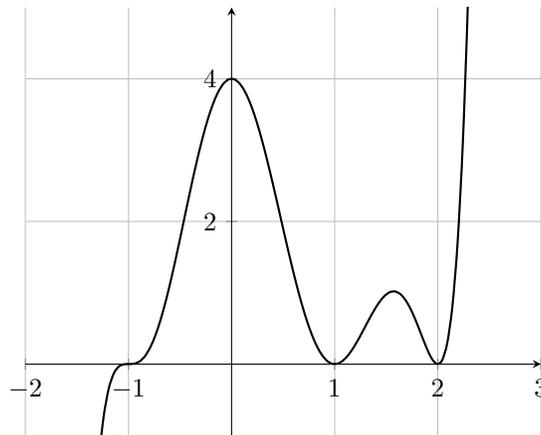
Identify the degree, the end-behaviour, the zeros and their multiplicities, and the y-intercept. Write down an equation of smallest degree for this polynomial.